PI Theory in Motor Control

This application note describes PI theory and PI in motor inverter control.

1 Introduction

1.1 Purpose
This application note describes PI in motor inverter control.

1.2 Definitions, Acronyms and Abbreviations
- P: Proportion
- I: Integral

1.3 Document Overview
The rest of document is organized as the following:
Chapter 2 explains the background of PI technical.
Chapter 3 explains the application in motor control.
Chapter 4 explains the application.

2 Technical Background

PI's theory

2.1 Overview
The PI loops use to faster and control motor.

Figure 1. Sample Module of PI Control

2.2 Theory

2.2.1 P - Proportion
The Proportional term of the regulator is formed by multiplying the error signal by a P gain, using the PI regulator to produce a control response that is a function of the error magnitude. As the error signal becomes larger, the P term of the regulator becomes larger to provide more correction. In short, P is a magnify function.
Figure 2. P Regulator

In figure 2, we can know the P regulator has two characteristic:

1. Timely and faster control.
2. When the adjust process finish, but still have the error signal.

The P regulator formula is:

$$u(t) = K_p e(t)$$  \hspace{1cm} (1)

Where $u(t)$ is output parameter, $K_p$ is the proportion coefficient, $e(t)$ is the input parameter.

2.2.2 I-Integral

The Integral term of the regulator is used to eliminate small steady errors. The I term calculates a continuous running total of the error signal. Therefore, a small steady state error accumulates into a large error value over time. This accumulated error signal is multiplied by an I gain factor and becomes the I output term of the PI regulator.

Figure 3. I Regulator

In Figure 3, we can know the I regulator have five characteristic:

1. Not timely and faster control.
2. If have error signal, the regulator output control will increase, and it’s speed always equal to start speed.
3. Control effect from strength to strength when time short and short.
4. I regulator can eliminate error signal.
5. In I regulator easy to bring adjust process surge.

The I regulator formula is:

$$u(t) = k_1 \int_0^t e(t) \, dt$$  \hspace{1cm} (2)

Where $u(t)$ is output parameter, $k_1$ is the integral coefficient, $e(t)$ is the input parameter, $t$ is the integral time.

2.2.3 PI

The Proportional and Integral are PI. PI regulator is used to produce a control response that is a function of the error until disappear. As the error signal generate, regulator produce a control response of P, the I regulator calculates the error value over time. So, the PI regulator has P and I regulator’s virtue, and eliminate themselves defect.
The PI regulator formula is:

\[ u(t) = u_p + u_i \]

\[ u(t) = K_p \left( e(t) + \frac{1}{T} \int_0^t e(t) \, dt \right) \]  \hspace{1cm} (3)

The PI regulator is a adjust module implemented with output saturation and with integral component correction. It can adjust input actual variable to follow the trace of target variable.

**Figure 5. Implementation Process of PI Regulator as a Adjust Module**

**INPUT** \( Y_{ref}, Y_{fb} \)
- \( Y_e = Y_{ref} - Y_{fb} \)
- \( U = X + K_p \cdot Y_e \)
- if \( U_{min} \leq U_k \leq U_{max} \)
  - \( U_k = U \)
- else if \( U_k > U_{max} \)
  - \( U_k = U_{max} \)
- else
  - \( U_k = U_{min} \)

**OUTPUT** \( U_k \)
- \( U_e = U - U_k \)
- \( X = X \cdot Z^{-1} + K_I \cdot Y_e + U_e \)
3 Application in motor control

PI regulator application in motor control

3.1 PI loop in FM3 inverter platform

Three PI loops are used to control three interactive variables independently, Speed PI loop, d-axis current PI loop, q-axis current PI loop.

![Figure 6. Three PI Loops Covered over Whole System](image)

In the figure 6, we can know the outer loop controls the motor velocity. The two inner loops control the transformed motor currents, I_d and I_q. The I_d loop is responsible for controlling flux, and the I_q value is responsible for controlling the motor torque. So we can use PI loop to control motor flux and torque.
3.2 Increment type PI loop in FM3 inverter platform

In FM3 inverter platform the PI loop type is increment type.

What is increment type PI loop? For example, define a array temp[10]={0,1,2,3,4,5,6,7,8,9 }, every time output the temp[k]= 0,1,2,3,4,5,6,7,8,9; Or output the temp[0], then output temp[1]=temp[0]+1,then output temp[2]=temp[1]+1 ……; it’s mean: this time output is last time output add the delta, this is the increment type PI loop.

Define:

\[ u[k] = u[k - 1] + \{u[k] - u[k - 1]\} \]

Then:

\[ u[k] = u[k - 1] + \text{delta} \]

Where: \(u[k-1]\) is last time PI output , \(u[k]\) is the PI output, delta is the PI error.

In math formula:

\[
\begin{align*}
  u_k &= k_p \left( e_k + \frac{T}{T_i} \sum_{j=0}^{k} e_j \right) \\
  u_{k-1} &= k_p \left( e_{k-1} + \frac{T}{T_i} \sum_{j=0}^{k-1} e_j \right) \\
  \Delta u &= u_k - u_{k-1} = k_p \left( e_k - e_{k-1} + \frac{T}{T_i} e_k \right) = k_p (e_k - e_{k-1}) + k_i e_k
\end{align*}
\]  

Where: \(e_k\) is k time sample error value.

- \(k_p\) is the proportion coefficient.
- \(k_i\) is Integral coefficient, \(k_i = k_p \frac{T}{T_i}\).
- \(u_k\) is the output value.
- \(\Delta u\) is the twice between border output error value.

4 Application

PI application achieve in system code

4.1 Function Description

```c
/* active statistic task number */

Function Name:       PI_Weakenl

C file Name:          PI.C, PI.H

Function interface: Q15_VAL32 PI_Weakenl(volatile_stPIDPara *pPI_Q15)
```
**Table 1. Input and Output of the Function**

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<td>Q15_VAL32</td>
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<td>e_Last1_Q15</td>
<td>Last error value input</td>
<td>Q15_VAL32</td>
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<td>Kp_Q15</td>
<td>P gain value</td>
<td>Q15_VAL32</td>
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<td>Ki_Q15</td>
<td>I gain value</td>
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<tr>
<td>Outputs</td>
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<td>Last output</td>
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<td>uMax_Q15</td>
<td>Output Integral limit maximal value</td>
<td>Q15_VAL32</td>
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<tr>
<td></td>
<td>uMin_Q15</td>
<td>Output Integral limit minimal value</td>
<td>Q15_VAL32</td>
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**4.2 Module usage**

The following code is example for this module.

```c
void example_PI_WeakenI ()
{
    Q15_VAL32 PI_output;
    pPI.e_Q15 = INa;
    pPI.e_Last1_Q15 = INb;
    pPI.Kp_Q15 = INc;
    pPI.Ki_Q15 = INd;
    PI_output = PI_WeakenI (&pPI);
}
```
## Document History

Document Title: AN205344 - PI Theory in Motor Control  
Document Number: 002-05344

<table>
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<tr>
<th>Revision</th>
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<td>**</td>
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<td>04/07/2011</td>
<td>Redraw some picture and add explanation</td>
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<td></td>
<td></td>
<td>06/08/2012</td>
<td>Changed the format</td>
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<td>5045172</td>
<td>CCTA</td>
<td>01/05/2016</td>
<td>Migrated Spansion Application Note from MCU-AN-510105-E-12 to Cypress format</td>
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<td>5701669</td>
<td>AESATMP9</td>
<td>04/19/2017</td>
<td>Updated logo and copyright.</td>
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