The Filter Wizard
issue 18: Gee, I see! The Ins and Outs of Generalized Impedance Converters
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OK, settle down at the back of the room, gentlemen and ladies! I know it made a nice change for you all that Jorge ran the last class while I was off doing battle with some hot transistors. Let him be an example to you all of what you can achieve when you actually do the darned homework. But I'm back, and there's more work to do to actually make some filters.

Filter Wizard #16,"Bruton Charisma", showed how we could apply an arbitrary scaling factor to each impedance in an LRC filter network, to create another network that has the same transfer function but is assembled from a different 'set' of components, with familiar Rs and Cs and also a new guy, the D-element. But we didn't get as far as figuring out how we would actually make up such a thing, a 'component' whose impedance equals -1/(Dw^2).

Such a component doesn't exist in passive form. How might we systematically develop a way of creating such a circuit configuration? Do we have to randomly juggle around a bunch of Rs, Cs and opamps until we encounter a circuit that has an impedance formula that does the job? Well, that's another Million Monkeys project in its own right; interesting work has been done on topology development by guesswork {cough} sorry, I mean optimization. The subject of a future article, perhaps, but not disciplined enough for my purpose here.

Here we'll address the central problem straight on. We've shown that we can **conceptually** create a new element by multiplying a component's value by a factor K that's a function of frequency. Can we now **practically** realize such an element by attaching our initial component to some kind of 'impedance scaler'? With such a scaler, indicated in figure 1(a), we could hang a certain component of impedance Z on one port, and when we look in on the other port, we'd 'see' an impedance of Z*K. Now, if K were a real constant, we already know how to do that, it's just a transformer with a turns ratio of sqrt(K). But that doesn't help us when K=1/(jw), or any other function of jw for that matter.

Well, it turns out that we **can** perform such impedance scaling on actual components; let's look at how we deploy key properties of the humble op-amp to do this. In all of this work, we'll investigate only the creation of scaled elements with one terminal connected to ground. Remember that in "Dualling Master" I gave advance warning that this would be much easier, or at least much more economical of parts, than generating truly floating scaled components.

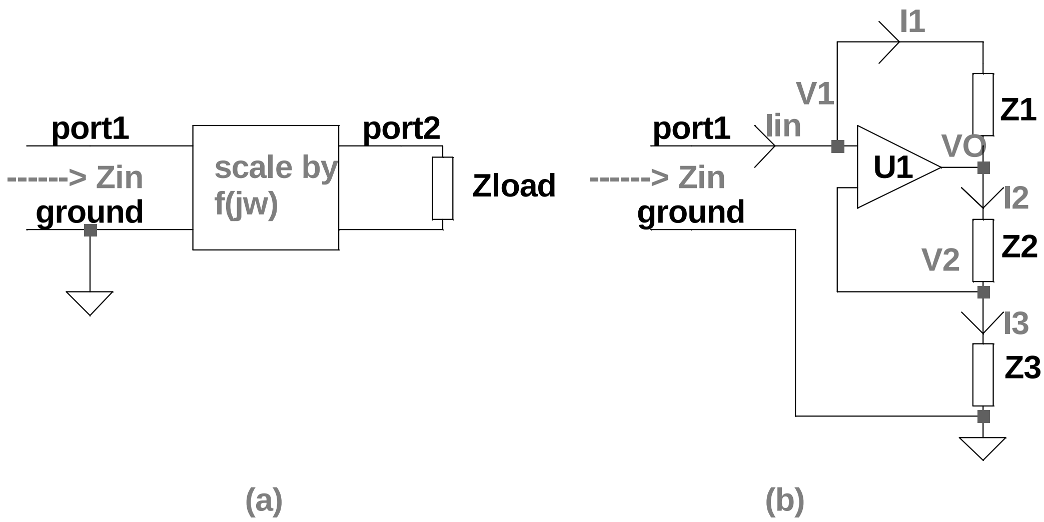

Figure 1: an 'ideal' impedance scaler, and an op-amp circuit to explore

Two key attributes of a good, high-gain op-amp when it's running linearly in a stable, closed-loop circuit are that (1) there's essentially no voltage between the two input pins and (2) there's essentially no current flowing into either input pin. So, consider the circuit shown in figure 1(b) (and yes, I know the input polarity symbols are missing from the amplifier, I'll get to that). Because of property (1) we can see that V2=V1=V, say, and because of property (2) we can write down by inspection

$$I_{in} = I_1 = \frac{V - V_O}{Z_1} \text{ and } I_3 = I_2 = \frac{V_O - V}{Z_2}$$

and thus

$$I_{in} = \frac{-I_3 \cdot Z_2}{Z_1}$$

and because $I_3 = \dfrac{V}{Z_3}$

we have $I_{in} = \dfrac{-V \cdot Z_2}{Z_1 \cdot Z_3}$

$$\text{and } \boxed{Z_{in} = \frac{V}{I_{in}} = \frac{-Z_1 \cdot Z_3}{Z_2}} \qquad eqn.1$$

This looks promising, because of course our Zs don't have to be resistive; they can have some frequency dependence. But before you get too excited I need to tell you that this **isn't** our 'Holy Grail' circuit yet, for a couple of good reasons. Firstly, there's **no** combination of Rs and Cs we can use for Z1 through Z3 that quite leads to the correct impedance function for our D-element.

It gets worse. Remember I left out the polarity signs for the op-amp? That was to emphasize that we can write down the circuit equations that lead to eqn.1 without knowing the polarity! But, remember the caveat for the initial conditions: this has to be a **stable** circuit. Clearly this circuit has both positive and negative feedback, in amounts that depend not only on Z1 through Z3 but also on the impedance of whatever is connected to terminal 1 (and also, obviously, on which way round we connect the amplifier). Any combination of these impedances and choice of feedback phase that cannot deliver stability cannot be used as a practical circuit. One usage configuration that does crop up occasionally is figure 2, which realizes a negative capacitance – note that this is only usable where such a component wouldn't itself destabilize your circuit!

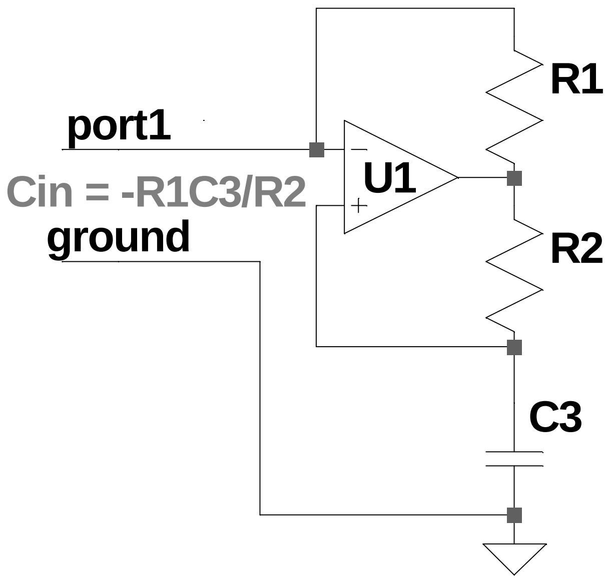

Figure 2: simple negative impedance converter realizing a negative capacitance

This circuit is a simple example of the class of circuits called Impedance Converters (it's a **negative** impedance converter, by virtue of the sign in eqn.1). The next step in our journey is to investigate a more elaborate two-op-amp configuration whose flexibility has earned it the term Generalized Impedance Converter, or GIC for short. The circuit we'll look at has become the most popular of the many theoretical ways to achieve this function, and is usually referred to as the Antoniou GIC [ref.1].

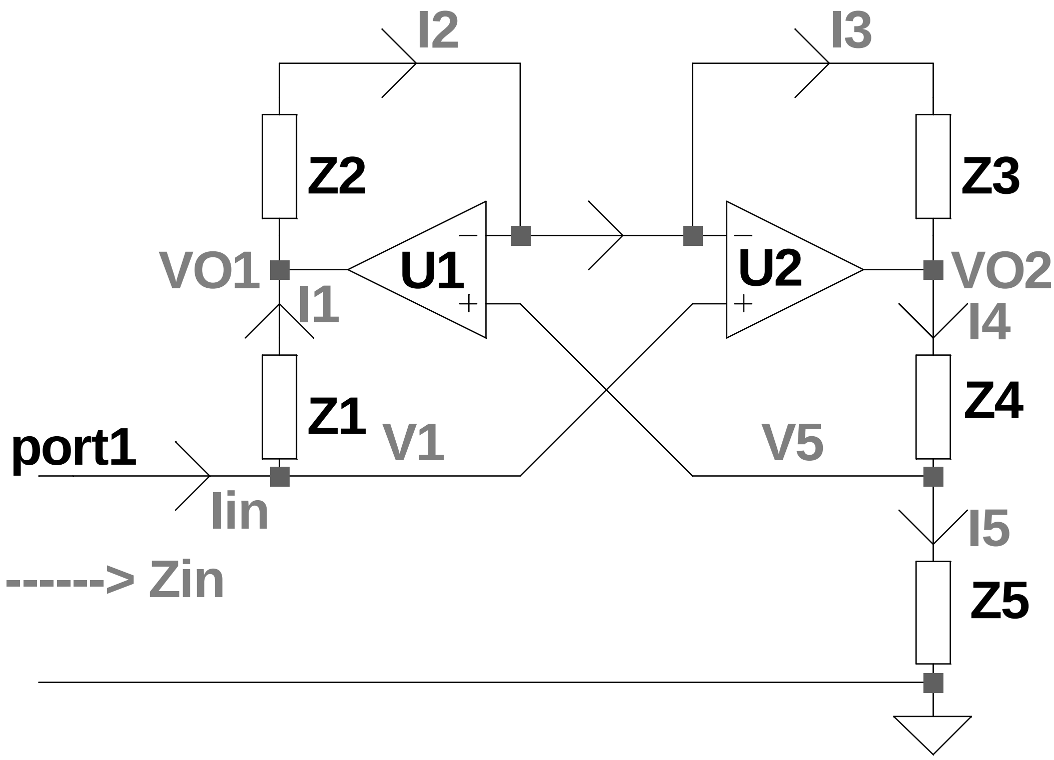

Figure 3: a Generalized Impedance Converter

Consider figure 3's circuit; it's two of our simple impedance converters butted back to back and cross-coupled to the opposite output. With the connection topology and polarity in the figure, it can be shown that as an impedance converter the block is unconditionally stable. What that means is that if the impedance you want (including non-physical ones such as the D-element) would theoretically be stable when embedded in the rest of the network you're making, the synthesized implementation with this topology will be too [ref. 2]. It's straightforward to follow the same analysis sequence as we used on the simpler circuit (why not have a go?). Remember, all four amplifier inputs are at the same potential. Here, I'll just jump to the key result:

$$\boxed{Z_{in} = \frac{V}{I_{in}} = \frac{Z_1 \cdot Z_3 \cdot Z_5}{Z_2 \cdot Z_4}} \qquad eqn.2$$

Note that figure 3's circuit is symmetrical in the sense that we could attach an impedance Z0 to the input port at Z1, and look into the port where previously Z5 was fitted. If Z0 isn't equal to Z5 then we'll 'see' a different impedance looking back like this. The circuit doesn't "look the same from both ends" and can't be used to create a floating component (which must inherently "look the same" either way).

Eqn.2 can be used to create our required D-element impedance expression in three ways. If we make Z5 a capacitor, to match the original capacitor in our filter before

Brutonization, then by making either Z1 or Z3 a capacitor and the rest resistors, we achieve an apparent impedance at terminal 1 that's exactly of the form that we need. We can think of the circuit as having done the required impedance transformation 1/(jw) on capacitor Z5. Another way is to make Z5 a resistor with both Z1 and Z3 as capacitors. In this case, we could think of having transformed the resistor directly by a term -1/w^2. Figure 4 shows two approaches. Really, you don't need to think of any particular component as actually being 'transformed'; that was just a useful conceptual route to arrive at the circuit configuration we'll use.

The frequency response at the internal nodes (VO1 and VO2) is different for each choice of component sequence (can you see why?). This influences both the overload margin and the noise spectrum, so it's always worth checking different variants during design.

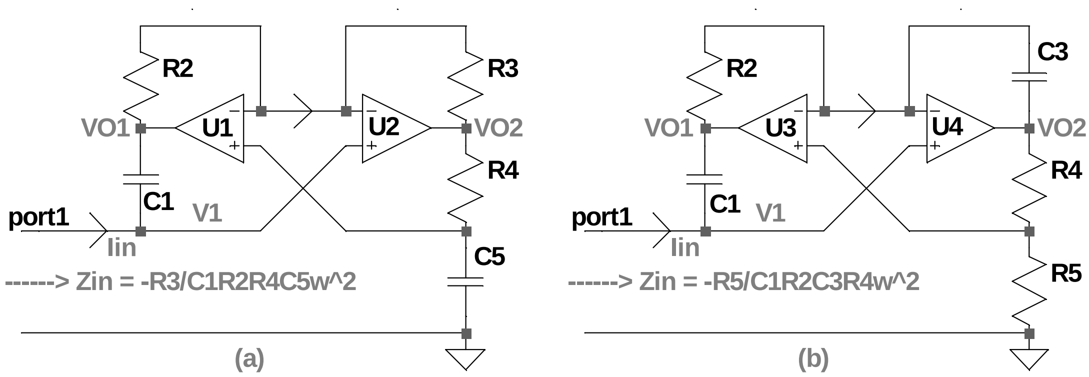

Figure 4: two ways of making a D-element with a GIC

We have a filter just waiting for us to impedance-convert, from "Bruton Charisma". I'll use the figure 4(a) configuration for the D-elements. Let's make the capacitors have the same value as the ones we're already using, that'll keep the production guys happy. Let's make Z2 a resistor of the same value as Z3 so that we can just cancel them out of the impedance equation. That then leaves Z4 to do the heavy lifting of defining the value of the D-element:

$$Z_{in} = \frac{Z_1 \cdot Z_3 \cdot Z_5}{Z_2 \cdot Z_4}$$

$$Z_1 = \frac{1}{j\omega C_1}, Z_2 = R, Z_3 = R, Z_4 = R_4, Z_5 = \frac{1}{j\omega C_5}$$

$$\therefore Z_{in} = \frac{R}{j\omega C_1 \cdot R \cdot R_4 \cdot j\omega C_5}$$

$$\therefore Z_{in} = \frac{-1}{\omega^2 \cdot R_4 \cdot C_1 \cdot C_5}$$

and for a D-element

$$Z_D = \frac{-1}{D \cdot \omega^2}$$

$$\therefore D = R_4 \cdot C_1 \cdot C_5$$

$$and\ R_4 = \frac{D}{C_1 \cdot C_5} \qquad eqn.3$$

In our example, it's easy to write down the values of the resistors needed to define the D-elements (RD2, RD4, RD6 below) by inspection from the "Bruton Charisma" circuit, since the product of two 10 nF capacitors is just 10^-16. Resistor value R is 'arbitrary', meaning you can choose any good engineering value. My experience with this variant of the circuit indicates that 1k ohm~2.2k ohm works well with regular op-amps, even when the remaining resistors in the circuit are much higher in value.

So, my friends, behold figure 5, the accumulated efforts of the Million Monkeys, the Dualling Master and the charismatic transformation of Mr Bruton, realized with Mr Antoniou's Impedance Converter.

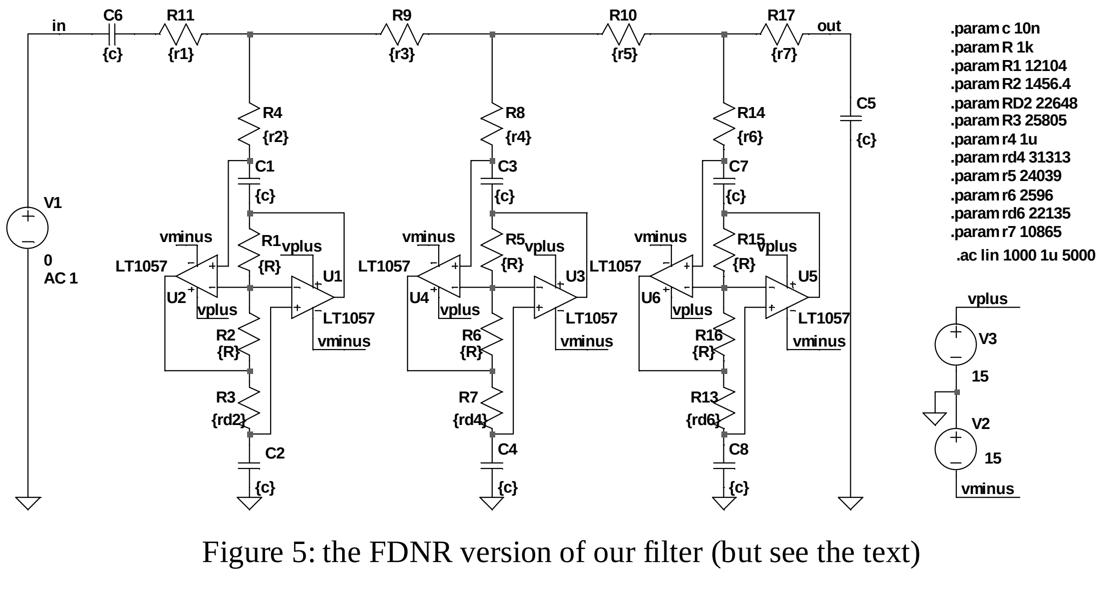

Figure 5: the FDNR version of our filter (but see the text)

At last, an active filter, whose response (figure 6) matches that of the passive circuit. Here I’ve used classic BiFET op-amps running on generous old-school +/-15V supplies. In my years as a commercial filter designer and manufacturer, we cranked out many thousands of channels of filters built just this way.

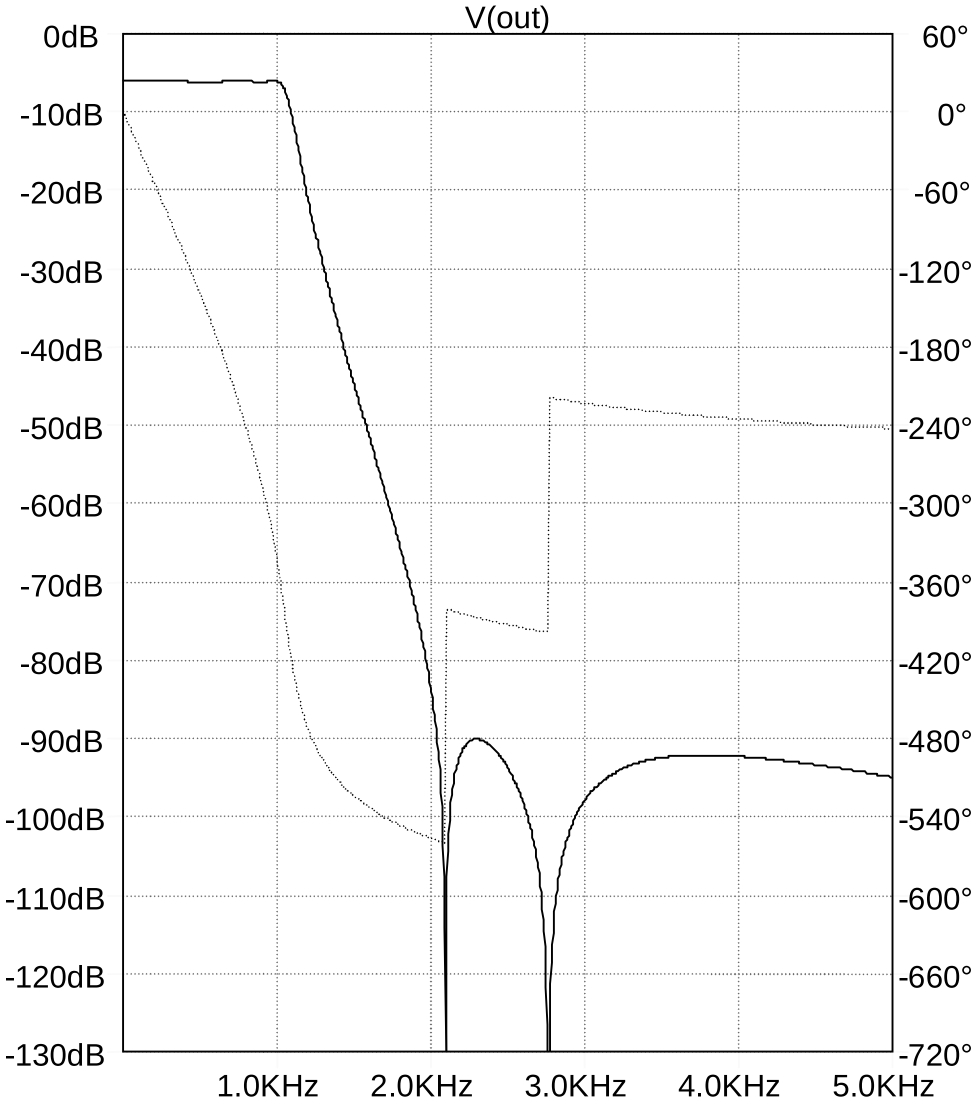

Figure 6: frequency response of figure 5 (but see the text)

Except there’s a catch. If you build the circuit shown in figure 5, it will stop working sooner or later (if it even starts in the first place). And if you try to simulate it with realistic models, you’ll have ‘tweak’ the simulation or it won’t run. This time I really do want you to have a look at this as homework and figure out what the problem is, and what I did (you can’t see it in figure 5) to get the simulation to run and produce figure 6.

The best explanation will get attributed in the next Filter Wizard, where we’ll find the problem, fix it (sort of), and look at a particular filter (designed for a particular project using the Cypress PSoC3) in which this problem can be turned to a useful advantage. We'll explore an obvious way (if you've studied earlier Filter Wizard pieces) of evading the problem, and that will point to yet more interesting developments to be covered in a future article. Until then, I hope this gets you to the "Gee, I see" stage <g> Happy impedance-converting, and don't forget to hand in your assignments to filterwizard@cypress.com! / Kendall

[1] "Realization of gyrators using operational amplifiers, and their use in RC-active-network synthesis", A.Antoniou, Proc. IEE, 116(11), 1838-1850 (1969)
[2] "Non-ideal Performance of Two-Amplifier Positive-Impedance Converters", L.T.Bruton, IEEE Trans. Circ. Theory, CT-14(4), 541-549 (1970)